Model-driven Sparse CP Decomposition for High-Order Tensors

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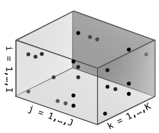
Background

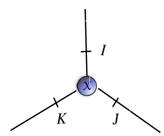
Tensor decomposition is a set of unsupervised methods to analyze and extract knowledge from tensors, which is widely used in healthcare analytics, image processing, machine learning, and social network analytics.

Sparse tensors

Many real-world tensors are hyper-sparse and have specific features. To discover useful knowledge, efficient sparse algorithms are critical to performance and scalability.



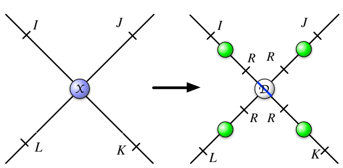
A 3rd-order sparse tensor



Tensor network diagram

CPD

CANDECOMP/PARAFAC decomposition (CPD) approximates an input tensor as a sum of component rank-one tensors, given the number of desired components. CPD is scalable in time and space compared to other low-rank methods.



CPD on a 4th-order tensor

Contributions

- This work proposes an adaptive and efficient algorithm for the computational kernel, Matricized Tensor Times Khatri-Rao Product operation (MTTKRP) of the classical CANDECOMP/PARAFAC decomposition (CPD), by minimizing redundant computations.
- We also build a model-driven procedure to determine the adaptable algorithmic parameters for different input sparse tensors and enable the trade-off between time and space based on the user need and memory resource.
- Our adaptive algorithm shows up to 10x speedup compared to state of the art on real-world high order tensors. Our method also shows near constant scalability with respect to the tensor order, while using acceptable storage space.

Algorithms

- Decouple an MTTKRP operation into a TTM and a TMHR.

Algorithm 3 Memorized Full MTTKRP algorithm (memFM).

Input: A fourth-order sparse tensor $\mathcal{X} \in \mathbb{R}^{I\times J\times K\times L}$, dense factor matrices $\mathbf{B} \in \mathbb{R}^{J\times R}, \mathbf{C} \in \mathbb{R}^{K\times R}, \mathbf{D} \in \mathbb{R}^{L\times R}$, the product order p_o;
Output: Updated dense factor matrix $\tilde{\mathbf{A}} \in R^{I\times R}$;
$\triangleright\ \tilde{\mathbf{A}} \leftarrow \mathcal{X}_{(1)}(\mathbf{D}\odot\mathbf{C}\odot\mathbf{B})$

// Compute as $p_o = [4,3,2]$.
1: Save $\mathcal{Y}^{(1)}$: $\mathcal{Y}^{(1)} \leftarrow \mathcal{X} \times_4 \mathbf{D}$ — TTM $\bullet$
2: Save $\mathcal{Y}^{(2)}$: $\mathcal{Y}^{(2)} = \text{TMHR}\left(\mathcal{Y}^{(1)}, \mathbf{C}\right)$ — TMHR $\blacksquare$
3: $\tilde{\mathbf{A}} = \text{TMHR}\left(\mathcal{Y}^{(2)}, \mathbf{B}\right)$ **return** $\tilde{\mathbf{A}}$;

1st MTTKRP $\quad \tilde{\mathbf{A}} \leftarrow \mathcal{X}_{(1)}(\mathbf{D}\odot\mathbf{C}\odot\mathbf{B}) \Leftrightarrow \tilde{A} \leftarrow \underbrace{\underbrace{\mathcal{X} \bullet \mathrm{D}}_{\mathcal{Y}^{(1)}} \blacksquare\, \mathrm{C}}_{\mathcal{Y}^{(2)}} \blacksquare\, \mathrm{B}$

Algorithms

- Design a simple memoization algorithm for MTTKRP sequence.

$$\tilde{\mathbf{A}} \leftarrow \mathcal{X}_{(1)}(\mathbf{D}\odot\mathbf{C}\odot\mathbf{B}) \Leftrightarrow \tilde{A} \leftarrow \underbrace{\underbrace{\mathcal{X} \bullet D}_{\mathcal{Y}^{(1)}} \blacksquare\, C}_{\mathcal{Y}^{(2)}} \blacksquare\, B$$

Reuse degree: 2

$$\tilde{\mathbf{B}} \leftarrow \mathcal{X}_{(2)}(\mathbf{D}\odot\mathbf{C}\odot\tilde{\mathbf{A}}) \Leftrightarrow \tilde{B} \leftarrow \mathcal{Y}^{(2)} \blacksquare\, \tilde{A}$$

$$\tilde{\mathbf{C}} \leftarrow \mathcal{X}_{(3)}(\mathbf{D}\odot\tilde{\mathbf{B}}\odot\tilde{\mathbf{A}}) \Leftrightarrow \tilde{C} \leftarrow \mathcal{Y}^{(1)} \blacksquare\, \tilde{B} \blacksquare\, \tilde{A}$$

$$\tilde{\mathbf{D}} \leftarrow \mathcal{X}_{(4)}(\tilde{\mathbf{C}}\odot\tilde{\mathbf{B}}\odot\tilde{\mathbf{A}}) \Leftrightarrow \tilde{D} \leftarrow \mathcal{X} \bullet \tilde{C} \blacksquare\, \tilde{B} \blacksquare\, \tilde{A}$$

One Group

- Design an **adaptive** memoization algorithm for MTTKRP sequence, to flexibly choose number of memoized MTTKRPs and reuse degrees.

$$\tilde{\mathbf{A}} \leftarrow \mathcal{X}_{(1)}(\mathbf{D}\odot\mathbf{C}\odot\mathbf{B}) \Leftrightarrow \tilde{A} \leftarrow \underbrace{\mathcal{X} \bullet D \blacksquare\, C}_{\mathcal{Y}^{(2)}} \blacksquare\, B$$

Reuse degree: 1

$$\tilde{\mathbf{B}} \leftarrow \mathcal{X}_{(2)}(\mathbf{D}\odot\mathbf{C}\odot\tilde{\mathbf{A}}) \Leftrightarrow \tilde{B} \leftarrow \mathcal{Y}^{(2)} \blacksquare\, \tilde{A}$$

Group 1

$$\tilde{\mathbf{C}} \leftarrow \mathcal{X}_{(3)}(\tilde{\mathbf{B}}\odot\tilde{\mathbf{A}}\odot\mathbf{D}) \Leftrightarrow \tilde{C} \leftarrow \underbrace{\mathcal{X} \bullet \tilde{B} \blacksquare\, \tilde{A}}_{\mathcal{Z}^{(2)}} \blacksquare\, D$$

Reuse degree: 1

$$\tilde{\mathbf{D}} \leftarrow \mathcal{X}_{(4)}(\tilde{\mathbf{B}}\odot\tilde{\mathbf{A}}\odot\tilde{\mathbf{C}}) \Leftrightarrow \tilde{D} \leftarrow \mathcal{Z}^{(2)} \blacksquare\, \tilde{C}$$

Group 2

- Decrease the operation number.
- Probably increase the storage space.

Tradeoff

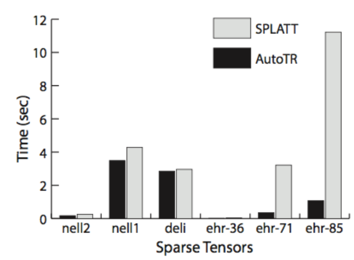
- Build a mode-driven auto-tuner to accurately predict the algorithm parameters for sparse tensors.

Results

We test our algorithm on Intel Xeon E7-4820 platform using 16 threads.

Dataset	Order	Max Mode Size	NNZ	Density
nell2	3	30K	77M	1.3e-05
nell1	3	25M	144M	3.1e-13
deli	3	17M	140M	6.1e-12
ehr-36	36	19	11K	4.7e-26
ehr-71	71	21	221K	1.4e-55
ehr-85	85	21	920K	7.9e-68

Sparse tensor description



CP-ALS runtime using SPLATT and AutoTR

Conclusion

Our optimization to increase the reuse of MTTKRP sequence is beneficial for high-order tensors. We use acceptable space to trade for higher performance.

References

B. W. Bader, T. G. Kolda et al., "Matlab tensor toolbox (Version 2.6)," Available online, February 2015.

S. Smith, N. Ravindran, N. Sidiropoulos, and G. Karypis, "Splatt: Efficient and parallel sparse tensor-matrix mul- tiplication," in Proceedings of the 29th IEEE Interna- tional Parallel & Distributed Processing Symposium, ser. IPDPS, 2015.

Acknowledgement: This project is supported by the U.S. National Science Foundation (NSF) Award Number 1533768.